The Irish Grid

A Description of the Co-ordinate Reference System used in Ireland

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INTRODUCTION

This paper describes the basis and derivation of the co-ordinate reference system used in Ireland. This system, generally known as the Irish Grid, is shared by the Ordnance Surveys of Ireland, based in Dublin and Belfast, and is widely used professionally throughout Ireland to describe positions on the earth's surface in an unique and unambiguous manner.

The availability of digital mapping and the potential for satellite positioning systems have increased many users interest in the Irish Grid and how it relates to other positioning systems - in particular the Global Positioning System (GPS). This is the first in a series of technical papers aimed at informing OS data users and the public in general alike on a number of technical matters. The previous publication 'Ordnance Survey Tables for the Transverse Mercator Projection of Ireland' (1971), is now withdrawn, and is replaced by the present paper.

Basic geodetic concepts are introduced, and an outline of the development of the Irish Grid given. Standard formulae and constants are provided in a technical data section, followed by worked examples of computing in the co-ordinate system.

Acknowledgements

Some of the formulae and examples are derived from the booklet 'The Ellipsoid and the Transverse Mercator Projection', published by OSGB as Geodetic Information Paper No 1, © Ordnance Survey (Great Britain), Southampton, 1995. The Ordnance Survey is grateful to Mr. F. Prendergast, Dublin Institute of Technology, for checking the example computations.

CONCEPTS

Introduction

This section provides an introduction to the geodetic concepts behind the Irish co-ordinate reference system. For a more detailed treatment, and the mathematics involved, the reader is directed to reference [1].

The Shape of the Earth

The earth is approximately a sphere, but with a varied and irregular surface. One of the basic aims of the science of geodesy is to determine the position of points on (as well as above or below) the earth's surface, in a way which is unique and unambiguous. Although no simple mathematical model exists to cope with the variations in the earth's true shape, historical attempts to define its size and shape discovered that it approximates to an oblate spheroid (that is a sphere slightly squashed at the poles), or an ellipse rotated about the semi-minor axis which is aligned to the axis of rotation of the earth (an *ellipsoid*). In fact, points on the earth's surface deviate by up to 9 km from the best fitting global ellipsoid.

The Geoid

So what is the true shape of the earth? The *geoid* is defined as a surface on which the earth's attractive (i.e. gravitational) forces are everywhere equal, i.e. a gravimetric equipotential surface. This may be visualised conceptually as a surface which coincides with mean sea level (imagining there was no land), where the effects of non-gravitational forces, such as tides, currents and metreological effects, are removed. However, there are local gravitational anomalies to this simplistic concept, due to land mass, noticed particularly in mountainous areas, and these can distort the shape of the geoid locally.

The geoid is of fundamental importance in determining positions on the earth's surface as most measurements are made with reference to this surface. For instance, heights are referred to mean sea level (which is effectively the geoid), and many measurement devices, such as theodolites, use gravity to determine directions. Furthermore, satellite systems operate within an environment directly influenced by gravity. The geoid is not a simple mathematical surface (although it can be modelled), but deviates by up to 100m from an ellipsoid, largely due to variations in gravity around the globe.

The Reference Ellipsoid

Because the ellipsoid is a good approximation to the shape of the geoid, and it is simple to define mathematically, it has been used in classical geodesy for over 200 years to provide a figure of the earth on which positions may be given in terms of latitude, longitude and height above the ellipsoidal surface. The ellipsoid thus used is termed a *reference ellipsoid*. As stated before, the shape of the geoid varies around the globe, therefore different sized ellipsoids have been used for different regions. Each is chosen to fit the geoid as closely as measurement technologies and computational abilities allowed at the time they were established. For example, an ellipsoid which provides a good fit of the geoid over the whole globe is not necessarily the most suitable for North America, and neither would be the most appropriate for Ireland (see Diagram 1 for an exaggerated depiction).

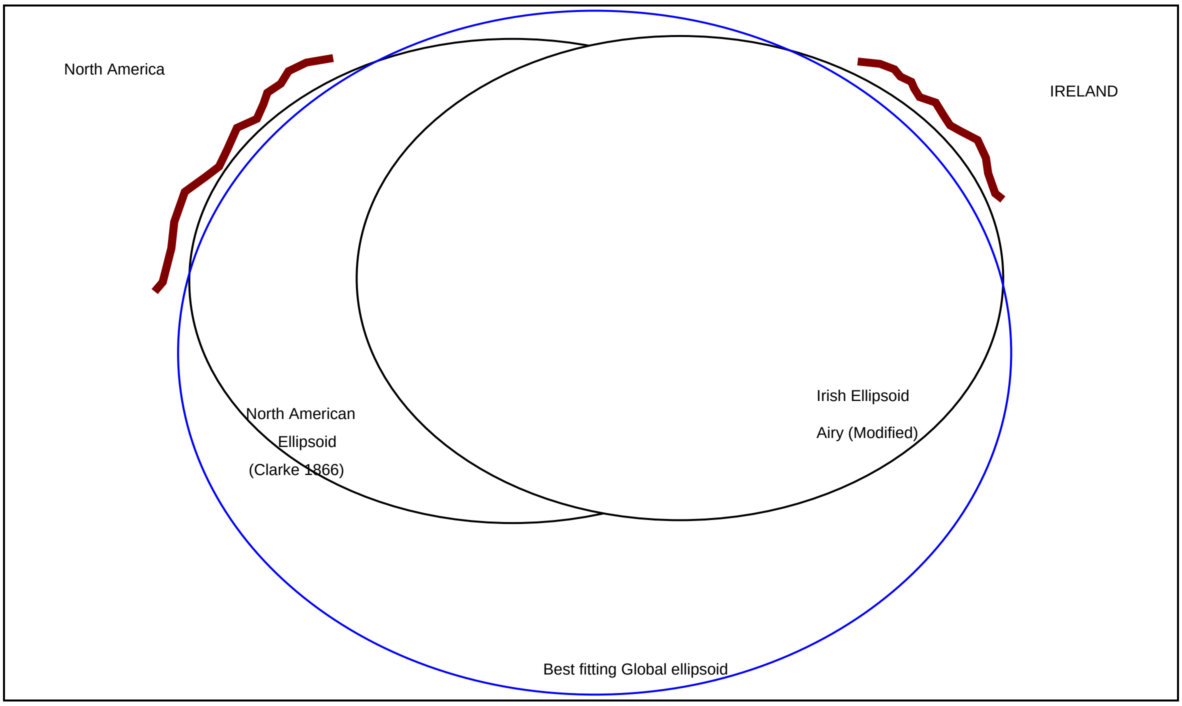

Diagram 1 ***: Exaggerated diagram of regional ellipsoids.***

This shows an imaginary section through the earth, with the regional ellipsoids positioned to closely fit the geoids of the country concerned. In a similar way, the best fitting global ellipsoid would not necessarily be the most appropriate for either region.

Geodetic Datum

Thus, there are many different ellipsoids on which positions may be expressed. The size, shape and positioning of the ellipsoidal reference system with respect to the area of interest is largely arbitrary, and determined in different ways around the globe. The defining parameters of such a reference system are known as the *geodetic datum*. The geodetic datum may be defined by the following constants:

- the size and shape of the ellipsoid, usually expressed as the semi-major axis (a) and the flattening (f) or eccentricity squared (e^2). There are a number of techniques used to determine the best fit ellipsoid for an area. Historically, triangulation was used in Britain and Ireland;

- the direction of the minor axis of the ellipsoid. This is classically defined as being parallel to the mean spin axis of the earth, and achieved by comparing the observed astronomic bearing of a line (say in a triangulation) with its calculated ellipsoidal bearing, satisfying the *Laplace*[1] condition, and adjusting the triangulation network as appropriate;

- the position of its centre, either implied by adopting a geodetic latitude and longitude (ϕ, λ) and geoid / ellipsoid separation (N) at one, or more points (datum stations), or in absolute terms with reference to the centre of mass of the earth;

- the zero of longitude (conventionally the Greenwich Meridian).

The manner in which the Geodetic Datum is defined varies from country to country (or region to region), usually through survey observation's, adoption of international standards, or acceptance of some form of historical convention.

Height Datum

Because the geoid is an irregular shape, it's surface is not generally parallel to the ellipsoidal surface. Therefore it is usual to fix the geoid at one location, usually some reference mark at which height above mean sea level has been determined, and refer heights to this point for practical purposes - this is known as the *height* or *vertical datum*.

Although the relationship between the geoid and ellipsoid is known at this point, and may be known at certain other points, the separation is not constant and furthermore can vary considerably, depending upon the nature of the geoid in the area of interest. Therefore some model of the variation may be required in order to determine the separation elsewhere. By choosing the best fitting ellipsoid this separation can, in certain circumstances, be ignored. However, with global ellipsoid's, or in areas of significant terrain variation, the separation and variation can be significant, particularly when transforming positions between reference systems. In these circumstances a geoid model is important.

Ellipsoidal Reference System

In this way the position of a given point (P) may be measured on the earth's surface and given a latitude (ϕ) and a longitude (λ) in terms of the reference ellipsoid (see diagram 2) by projecting it onto the ellipsoid's surface along a line in the direction which is perpendicular to the ellipsoid surface (the *normal*) - see diagram 3. It should be noted that the direction of the true vertical (direction of gravity) may be slightly different from the normal, and this difference in direction is termed the *deviation of the vertical***,** which for most practical purposes may be ignored. The distance along the normal to the ellipsoid is termed the ellipsoidal height (usually designated by the letter *H*), and fixes the point in 3 dimensional space. As noted before, the height above sea level (in geodesy termed the orthometric height, and designated by *h*) is the most useful height for practical purposes, and is usually measured by spirit levelling. The separation between the ellipsoid and geoid along the normal is generally known as the *geoid-ellipsoid separation*, (designated conventionally *N*), and is an important element in the computational process. Thus, positions are given in terms of latitude, longitude and ellipsoidal heights.

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[1] Laplace points provide an independent determination of ellipsoidal bearings, which may be then compared to the computed bearing to determine any errors in the angles used to carry bearings forward through a survey network. Classically it is usual in geodetic networks to include such points in the final adjustment.

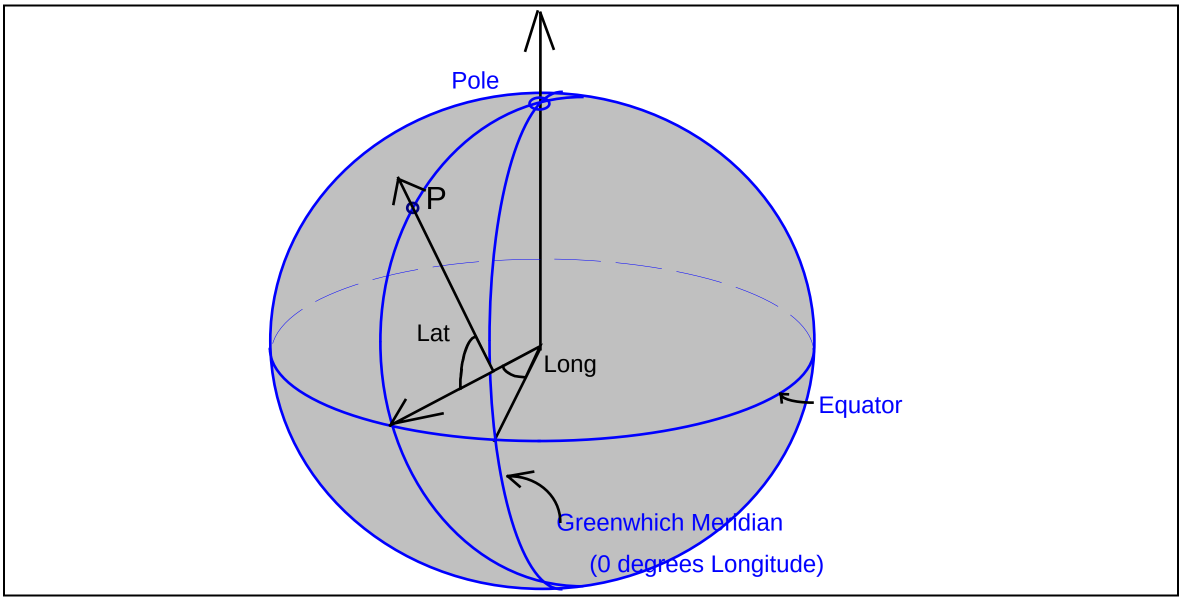

Diagram 2 : *Ellipsoidal Reference system.*

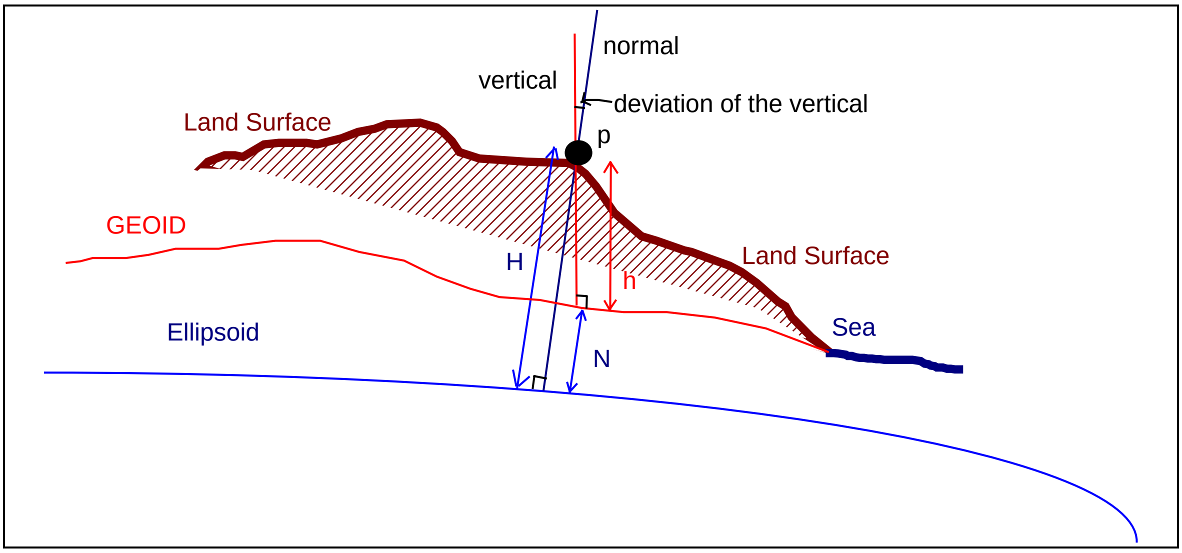

Diagram 3 : *The relationship between the ellipsoid, the geoid, land surface, normal and true vertical*

Cartesian Reference System

Positions may be given in absolute terms, relative to the earth's centre of mass, or an assumed centre (as implied by a geodetic datum). In this system a position is defined in 3 dimensional space by an X, Y, Z co-ordinate triplet, with the Z axis passing through the centre of the earth (or reference ellipsoid) and the poles, the X axis through the centre and the Greenwich meridian, and the Y axis at right angles to these. Other parameters may define this system, but are not directly relevant here.

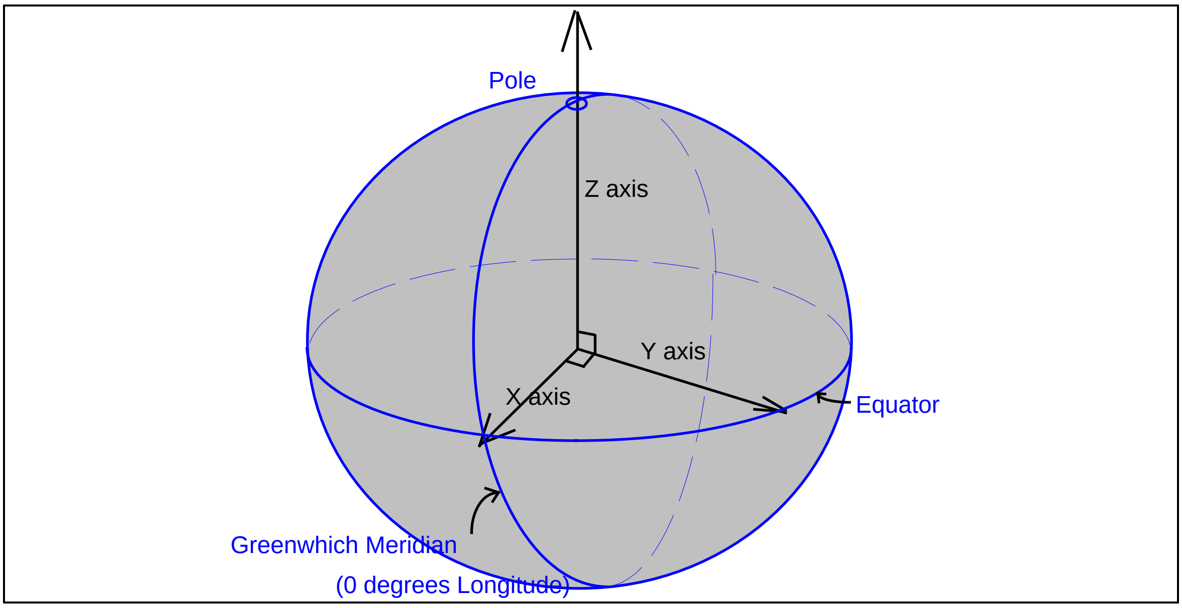

Diagram 4 : *Cartesian Reference System.*

It is important to realise that the centre of a Cartesian reference system for an adopted ellipsoid may not be the 'true' or adopted centre of mass of the earth - the latter is often used for global reference systems (as used by the Global Positioning System, GPS). Furthermore, the direction of the Z axis may differ. This has led to one of the major requirements of geodesy and surveying today, which is how to relate global to local referencing systems so that positions in one may be expressed in terms of the other (and vice-versa). This issue is not dealt with here, except to state that it is necessary to move from the ellipsoidal system to the local cartesian reference system before applying some translations in X, Y and Z, and possibly scale change and rotations around these axes as well, to move from the local system to global.

The cartesian reference system is particularly useful as calculations are simpler to perform (no knowledge of spherical geometry being required). However, the relationship between positions on the earth's surface are difficult to visualise in this system, and this is particularly important in navigation (for instance); the concept of height is also unclear.

Plane Co-ordinates

Having established a co-ordinate reference system, it remains to depict the position of points of interest onto a flat surface, e.g. a piece of paper. In general, it is usual to determine the position of some reference points in terms of the ellipsoidal reference system by measurement (such as triangulation), before projecting these onto a plane co-ordinate system and carrying out the survey of topographical detail in this simpler system. Projecting co-ordinates from a curved surface (latitude, ϕ and longitude, λ) on to a plane surface will cause some distortion, but this is minimised by choosing the most appropriate projection for the area concerned, and the features to be depicted.

There are many projections which can be used, and these may be seen in any world atlas. However, because the areas to be projected in a world atlas are large the distortions are unavoidable and clearly evident. Engineering and cadastral maps are generally of smaller areas depicted at scales larger than 1:50 000, and as these forms of map are often the basis for further measurement, it is necessary to keep distortions to a minimum. The property of retaining shape and scale is known as *orthomorphism*, or *conformality*.

The Transverse Mercator Projection

One projection with the properties of orthomorphism is known as the *Transverse Mercator*, or *Gauss Conformal* projection. Although conceptually simple, the mathematics involved are less so (in order to ensure orthomorphism). The basic formulae are given later in this paper.

A cylinder of a specified radius is wrapped around the reference ellipsoid so that its circumference touches the ellipsoid along a chosen meridian (line of longitude); (see diagram 5). The scale of the projected area is therefore correct along the chosen meridian. The radius of the cylinder will match the radius of the reference ellipsoid at a specified point. This provides the *Origin* of the projection (i.e. the line of longitude at which the cylinder touches, and the latitude of the reference ellipsoid where the radii match). The cylinder is then 'unwrapped', providing the flat surface of the map. The origin is generally chosen to be central to the area of interest, so that the distortions away from the origin are minimised.

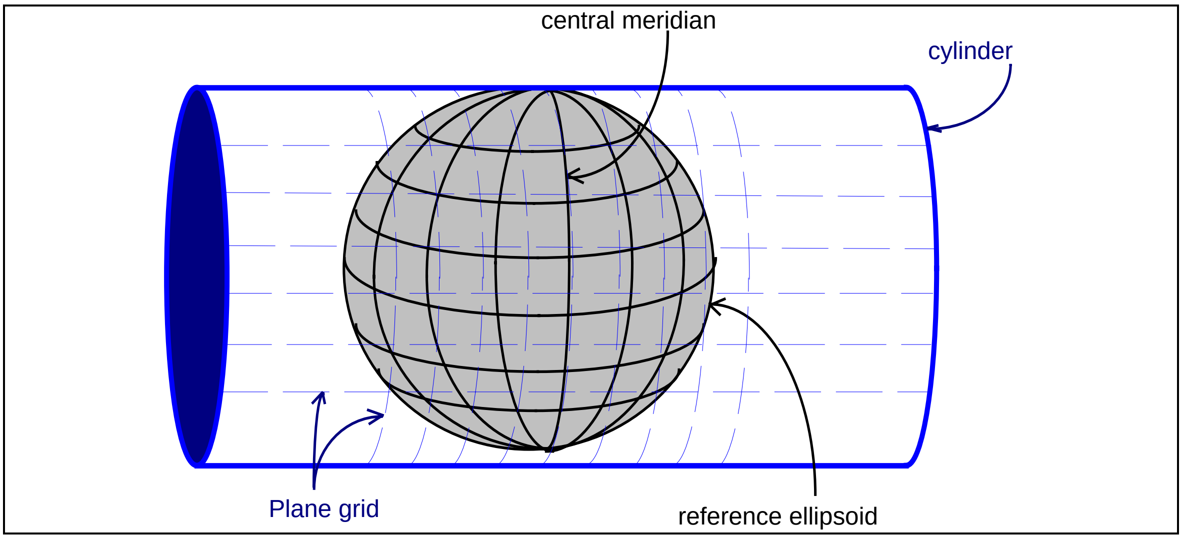

Diagram 5 : *Concept of Transverse Mercator Projection*

Scale increases away from the origin in an uniform way, so that scale at any point on the grid away from the central meridian (where it is uniform) may be corrected by applying a *scale factor* to determine true scale[2]. In this way, for short lines, true ground distances may be obtained by measurement off the map.

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[2] Implementations of the Transverse Mercator Projection can vary; for instance, in Great Britain the scale factor of the central meridian is reduced from 1.0 to 0.9996 approximately, reducing the size of the scale factor to be applied at the extremes of the area mapped.

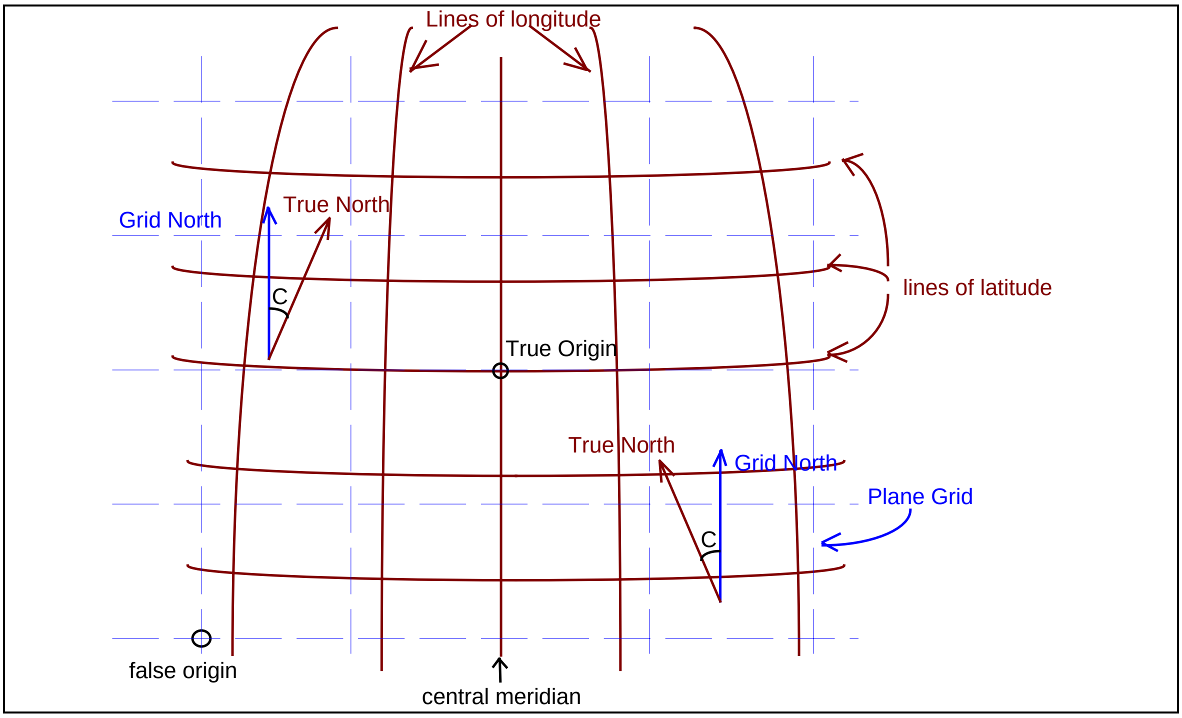


Diagram 6 : *Lines of latitude and longitude projected onto the plane grid reference system using the Transverse Mercator Projection, and Convergence,* C.

In order to avoid negative co-ordinates within the area of interest (as the origin will have plane co-ordinates of 0 in eastings and 0 in northings) it is usual to add a constant value to eastings and northings so that all plane co-ordinates are positive. This creates a *false origin* for the projection.

As lines on the reference ellipsoid project on to the plane grid as curved lines, two further corrections are required for precise computations. Along the central Meridian true north (the direction of the north pole) and grid north are the same. However, away from the central meridian the direction of true north differs by an amount which increases the further from the central meridian the point of interest is. This difference is known as *Convergence*, and it is the angle between true north and grid north - see diagram 6. Because straight lines on the reference ellipsoid project on to the plane grid as curved lines, known as *geodesics*, a further correction to a direction at a point is required. This is known as the *Arc to chord*, or *(t-T) correction* and it is the difference in angle between the initial direction of the curved geodesic and the straight line grid bearing. Geodesic curves are concave towards the central meridian. In diagram 7 (which is greatly exaggerated) two directions either side of the central meridian are depicted. The straight line A B represents a line between two points as plotted on the map, whose direction is termed t. The curved line A B represents the line of sight between the same two points in nature - this projects on to the map as a curved line (the geodesic), whose initial direction is termed T. The direction of the curved line A B at A is represented by the line A X, and it's initial direction is the tangent of the curve at A. It should be noted that the Transverse Mercator projection is conformal, therefore directions on the projection are relatively correct, and this correction is required only when comparisons with the real world are necessary, and only if one is interested in a few seconds of arc.

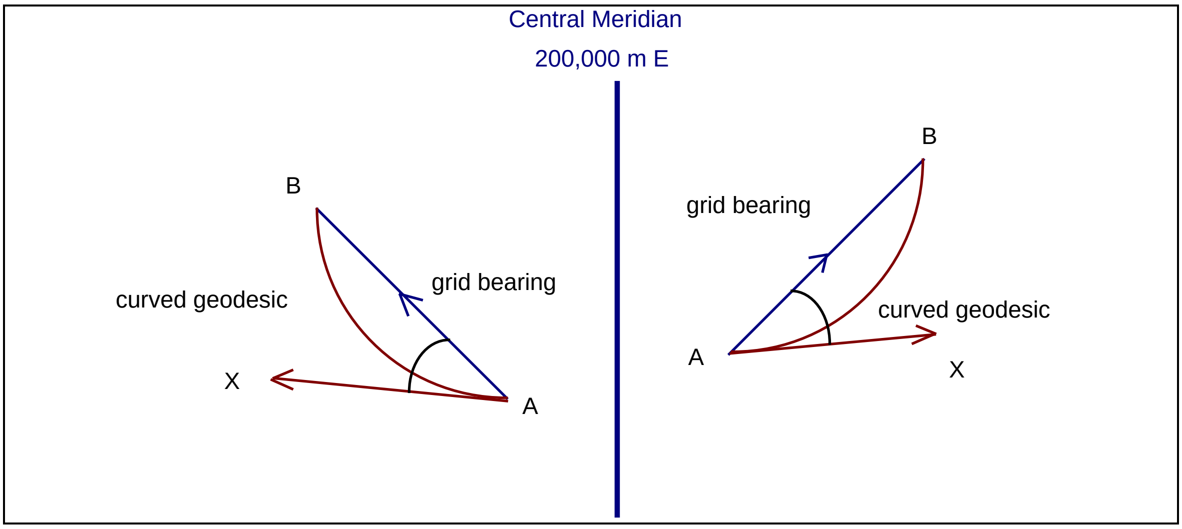


Diagram 7 : *The (t-T) correction*

Historical Context

Introduction

The Irish Grid has developed over more than two hundred years, in line with the development of scientific thought, measurement techniques and computational power. In recent times the development and wide use of new technologies, such as GPS and increasingly powerful computers, has highlighted some of the shortcomings of reference systems in use all over the world. This section provides a summary of the historical development of the Irish Grid. A fuller account may be obtained from the references given at page 35.

The Principal Triangulation of Great Britain and Ireland

The Principal Triangulation was originally begun in 1783, to determine the difference in longitude between the observatories of Greenwich and Paris. Following the establishment of the Ordnance Survey in 1791 it was gradually extended to cover the whole of Great Britain and Ireland, with observations completed by 1853. The triangulation approach adopted was necessary in order to transfer distances from the measurement of a base line on Salisbury Plain across the length and breadth of Britain and Ireland.

In 1824 the Spring Rice Committee recommended to the British House of Commons that a survey of Ireland at the scale of six inches to the mile was required to provide a definitive indication of acreages and rateable values for the purposes of establishing local taxes in Ireland. This task was begun by the Ordnance Survey on 22 June 1824.

The Principal Triangulation in Ireland commenced shortly after the Spring Rice Committee, and was completed by August 1832. It was not until 1858, however, that A.R. Clarke, who was then in charge of the Trigonometrical and Levelling Department of Ordnance Survey, had selected the observations to form the interlocking network of well conditioned triangles that is now known as the Principal Triangulation (see Diagram 8), rigorously adjusting it by the method of least squares in 21 independently computed, but connected, blocks, [4].

Bearing in mind the early date of many of the observations and the primitive nature of the instruments used, the results from the Principal Triangulation were impressive. The average of triangular misclosures was 2.8 seconds of arc, and the distance of the Lough Foyle base as computed through the triangles from the Salisbury Plain base was within 5 inches of it’s measured length [5].

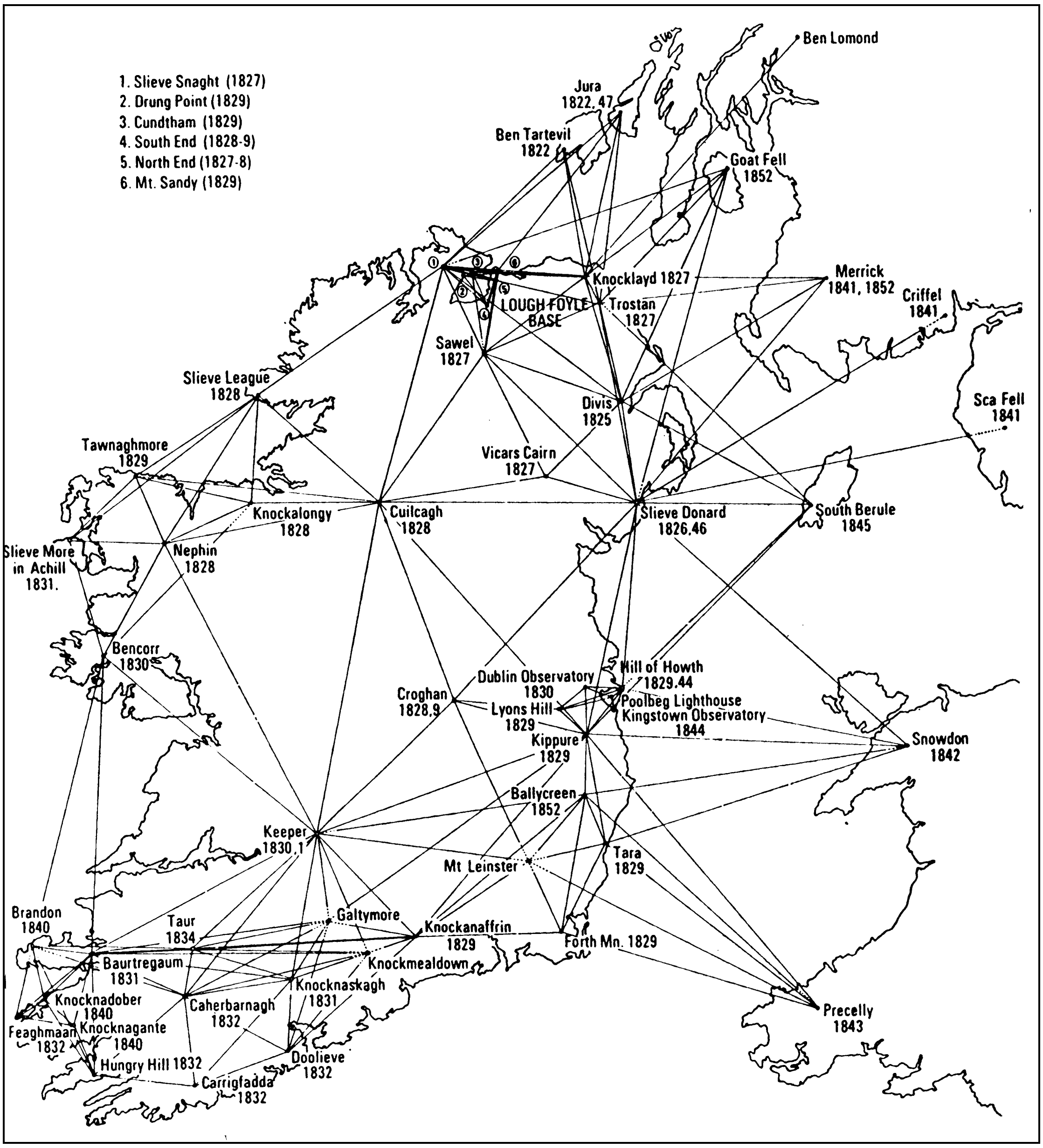


Diagram 8 ***: The Principal Triangulation of Ireland 1824 - 1832***

Early Map Control

The Principal Triangulation was not designed as a comprehensive national system to control mapping, but as a geodetic framework initially to determine the difference in longitude between Paris and London and later to define a figure for the earth.

Six Inch Map Control
The results of the Principal Triangulation adjustment were not available until the late 1850's [4]. However, some form of framework was required on which to control the new mapping at the scale of six inches to one mile. Six inch map control was therefore based on a network of secondary and tertiary 'blocks' of triangulation, begun in 1832 and completed in 1841, just ahead of the chain survey teams who were surveying the detail. Although these lower order triangulations may have included some of the Principal Triangulation points they were probably based on provisional values only. Furthermore, the blocks were computed independently of each other and by a variety of methods. This was expedient at the time, their purpose being for mapping on a county by county basis only. However, the result was that little sympathy existed between adjacent blocks of map control, which together with spherical reference systems used, independent county meridians and their associated Cassini projections caused discrepancies of up to 50 ft. between detail across adjoining counties [5].

One Inch Map Control
The one inch map, begun in 1852, was to be on a single national datum and projection, and compiled from the detail surveyed for the six-inch maps. Again the results from Clarke's adjustment were not available, and the method chosen to relate the county datums to the national one did not fully connect to the Principal Triangulation [5]; there were only five common points between it and the 32 county datums.

25 Inch Map Control
October 1887 saw Treasury approval granted for the 1:2 500 scale mapping of Ireland, and the work began in 1888. Initial methods involved replotting from the six-inch field books at the larger scale, but this was subsequently abandoned following the completion of Counties Down and Limerick in favour of a resurvey based on new secondary and tertiary triangulation. However, as before, secondary blocks were adjusted independently of each other and tertiaries by semi-graphical methods and although Clarke's adjustment was now complete, it was not used, and few records survive today of the work undertaken. This task was largely completed by 1913, and the results thus derived remain the basis of large scale maps for many parts of Ireland today.

Whatever may be said today about the methods of control chosen, they were adequate for the purposes they served at that time, and, *'When the map of Ireland is picked up and shaken, it is only the mathematician who hears the rattle'* (Andrews, [6] p.233). This 'rattle', however, is of more relevance in today's age of electronic computers, global geodetic frameworks and mapping across Europe than it was in the 1850's, therefore in recent times a number of actions have been taken to improve the situation.

The Re-Triangulation of Northern Ireland

After the Ordnance Survey of Northern Ireland was set up (1921), the Ordnance Survey of Great Britain retained responsibility for the geodetic triangulation until the end of the second World War. Priorities clearly lay elsewhere between and during the World Wars, and economic conditions did not help, therefore little action was taken to improve the condition of the geodetic framework or the mapping until after World War II.

Following the completion of the re-triangulation of Great Britain [5] (itself delayed due to World War II), resources were made available to the Ordnance Survey of Northern Ireland and observations on the re-triangulation began in the Spring of 1952. The network consisted of 9 stations, plus 3 in the Republic and a number of cross-channel connections to Great Britain (see Diagram 9).

The adjustment accepted the position of three of the original Principal Triangulation points in order to scale and orientate the new triangulation. One of these points was fixed to Clarke's original value (at Divis) and two (Knocklayd and Trostan) to values from Wolf's 'Mathematical Basis' (unknown reference) which varied slightly from Clarke's [7]. Comparisons with the (then) recent re-triangulation of Great Britain showed only a slight discrepancy between the two re-triangulations.

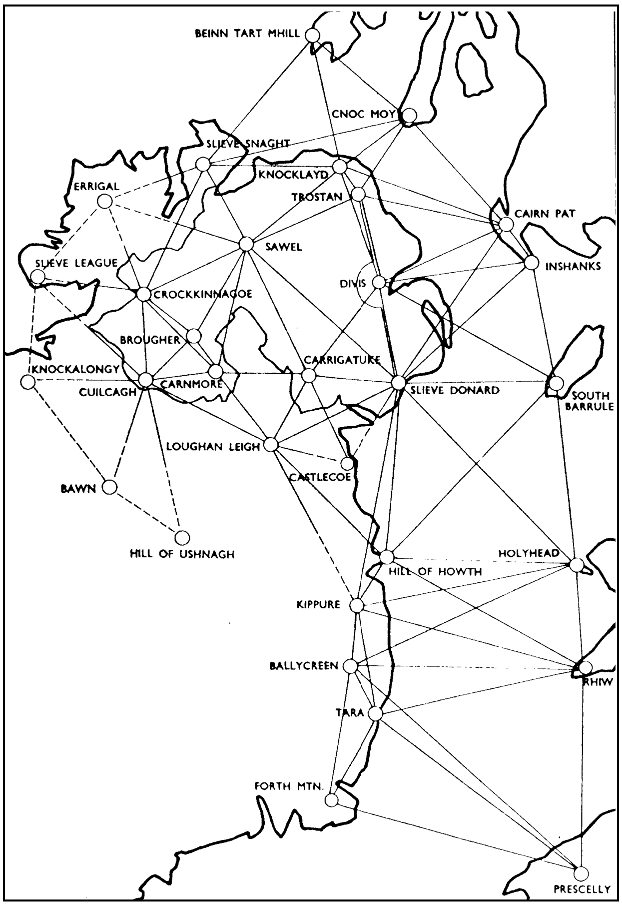


Diagram 9 ***: The Re-Triangulation of Northern Ireland 1952***

Mapping Control in Northern Ireland.

As soon as the Primary Triangulation was completed the Secondary Triangulation was re-commenced (one block having been completed earlier). This work finished in July 1956, and each secondary block was adjusted separately to the primary and to adjoining secondary stations.

The Secondary Triangulation was further broken down into Tertiary Triangulation and connections incorporated to stations used for controlling the 25 inch mapping. This established the differences between the county-based datums and the Irish Grid, allowing the mapping to be 're-cast' on to the Irish Grid, within tolerances acceptable at the time [7]. These networks also formed the basis for re-surveys of some areas of mapping which were either too out of date or where significant accuracy problems were known to exist.

The Primary Triangulation of Ireland

The Ordnance Survey of Ireland carried out a first order Triangulation of Ireland between the summer of 1962 and late 1964, the whole island being adjusted as one, with the observations from the 1952 Re-Triangulation of Northern Ireland included (Diagram 10). This time the availability of new Electromagnetic Distance Measurement (EDM) equipment allowed distances over long lines to be determined quickly for the first time, and this was done by measuring the sides of a braced quadrilateral figure by Tellurometer EDM in the south west of Ireland, providing some independent scale checks on the triangulation.

Some constraints on the new scheme were necessary. The new adjustment had to restrict the movement of the northern primaries to within 0.25 m of their 1952 positions, a tolerance chosen so as not to affect the mapping already completed or recast on to the Irish Grid in Northern Ireland. Initially the positions of Slieve Donard and Cuilcagh were held fixed to their values from the 1952 adjustment, with tellurometer and cross channel observations excluded. This caused too large a movement of the Northern Ireland stations from their 1952 values (up to 1.2 m in one instance), probably because the 1952 values were originally based on the Principal Triangulation, which contained an inherent scale error of between 30 and 40 parts per million (ppm). Therefore a modified ellipsoid and projection was tried and found to improve matters, moving Northern Ireland stations by 0.5m from their 1952 values, but causing unacceptable shifts at the common stations in the south.

It was clear that as geographicals somewhere had to change a little there was no particular virtue in holding Donard, Cuilcagh and the azimuth between them exactly as they were, if small shifts there would improve matters elsewhere. This was tried, with both a normal and modified Airy elliposid. The result with the normal Airy ellipsoid gave a rms vector shift of 0.341 m, with a maximum of 0.573m at Donard itself, and marginally better results overall with the modified ellipsoid.

Thus the Airy ellipsoid was modified by 35 ppm, and a modified Transverse Mercator projection adopted. This gave an overall scale correction to conform to the tellurometer distances observed while retaining geographical and projection co-ordinates substantially unaltered, with the grid to graticule tables, used for conversion between the two, completely unaltered. This latter fact was of course very important before the days of electronic calculators and computers.

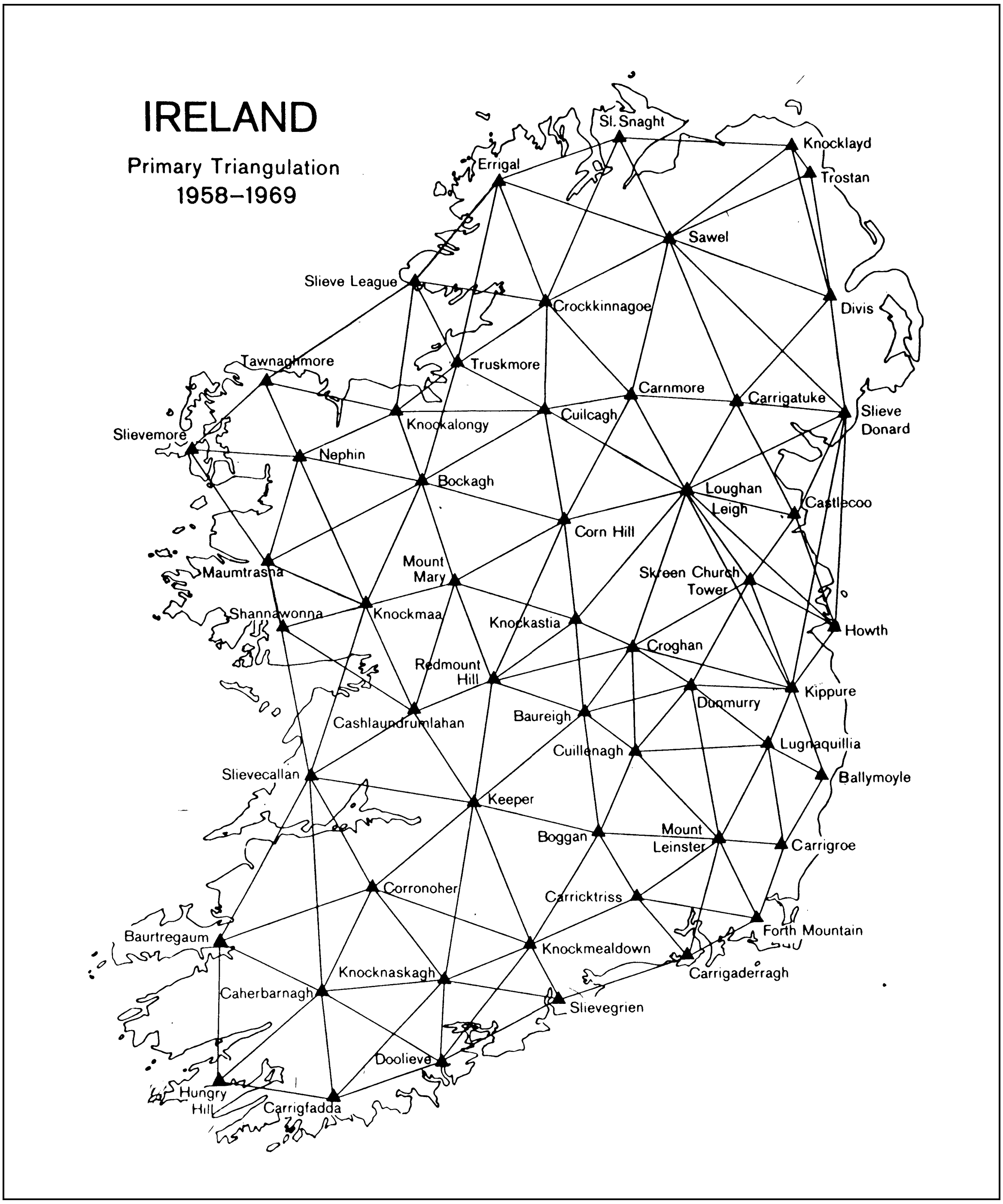


Diagram 10 : ***The Primary Triangulation of Ireland.***

The results from this adjustment are summarised in the next table:

Root Mean Square correction to an observed angle	± 0".96
Maximum adjustment correction	2".18
Average angle correction due to scale factor	0".002
Root Mean Square Error (RMSE) of observed *v* computed distances	± 7 ppm
Maximum RMSE of observed vs computed distances	±12 ppm

A comparison with the 1952 adjustment gave root mean square changes of ± 0.25 m in Eastings and ± 0.23 m in Northings and in a maximum vector difference of 0.57 m.

Subsequent Laplace observations were carried out on two lines (Carnmore to Carrigatuke and Doolieve to Carrigfadda) in 1966, and thus were not included in the 1965 adjustment. However, these indicated that a mean rotation of the network by +2.27 seconds would satisfy the Laplace conditions. Comparison of the two lines showed only 0.5 seconds of arc difference between them, indicating very little internal distortion in the network, albeit a small rotation overall.

Further work in 1969 re-observed Tellurometer EDM distances across St. George's Channel and the Irish Sea, and observed Geodimeter EDM distances between four further Irish stations. These latter observations revealed a continuing scale error of +5ppm in the 1965 adjustment.

1975 Mapping Adjustment

Ordnance Survey Belfast did not adopt the 1965 values for subsequent mapping, whereas Ordnance Survey Dublin did. To address the problem of large scale maps meeting at the border a re-computation of the 1952 and 1962 observations was begun. However, many parts of Ireland had already been mapped, therefore a major restraint on this new adjustment was to prevent any plottable shift in those areas already mapped. (i.e. Northern Ireland, Dublin and Cork). Thus all Northern Ireland primary stations were held fixed at their 1952 values and three primaries in the Republic (Howth, Kippure, and Doolieve) were fixed at their 1965 values. Only angle observations were used in the adjustment, which is known as the 1975 Mapping Adjustment, and the resulting shift from the 1965 adjustment was as follows:

Average difference in Eastings	0.092 m
Average difference in Northings	0.108 m
Maximum vector difference	0.548 m

The Airy Modified Ellipsoid was again used, and the datum (known as the 1965 datum) is derived from the values of the stations held fixed in the adjustment.

Mapping Control

1:1 000 Map Control
By 1975, secondary and tertiary blocks had already been completed in Dublin and Cork to provide control for the new urban re-survey at 1:1 000 scale. In order to relate the 1965

values of these stations to the 1975 Mapping Adjustment a transformation of co-ordinates was carried out using the primary stations in these blocks as a basis for calculating the transformation. The transformations were applied to the secondary networks only; the tertiaries retain their 1965 values. Other urban areas mapped at 1:1 000 scale were controlled from stations with 1975 mapping adjustment co-ordinates.

The secondary triangulation of the country was carried out in approximately 12 blocks. The first observed block in Dublin was completed by triangulation using Wild T3 theodolites and the remainder utilised EDM traversing (Tellurometer MRA3, CA1000 and Siemens). Each block was adjusted separately on to the 1975 adjustment using variation of co-ordinates, and the final block in Kerry was completed in 1986.

The tertiary order control was carried out in numerous blocks each of which was observed by tellurometer and theodolite, and adjusted separately to the 1975 adjustment (except for Cork and Dublin). The adjustments were carried out using variation of co-ordinates but the network was never completed. The final block in Donegal, due for completion in 1987, was not observed because the arrival of the Global Positioning System (GPS) provided an alternative method of providing mapping control.

1:2 500 (NPS) Map Control
Revision of mapping based on the original 1:2 500 County Series was halted during the 1980s, and a new National Photogrammetric Survey (NPS) begun. These surveys are currently based on the 1975 Mapping Adjustment network, using GPS.

IRENET'95

During the late 1980's and early 1990's the increased use of GPS for mapping and scientific work highlighted the need for a reference network compatible with the new technology and essential to relate the mapping framework to global and continental reference systems. An early attempt to do this based on three Primary stations was not a complete success due to faulty equipment and an immature satellite constellation. Therefore in April 1995 a new geodetic and survey control network (IRENET) was observed using the Global Positioning System (GPS). This network consists of 12 new 'zero-order' control stations (8 in the Republic, 3 in Northern Ireland and one on the Isle of Man) connected to some of the defining International Terrestrial Reference Frame (ITRF) stations in Europe - see diagram 11. The resulting adjustment was accepted as an official extension to the European Terrestrial Reference System (ETRS) by sub-commission X (EUREF) of the International Association of Geodesy (IAG) in Ankara, Turkey, 1996.

These zero order stations were used to control a densification of the network to a further 173 stations throughout Ireland between May and December of the same year. Co-ordinates have been computed in terms of ETRS89 and Irish Grid (1975 Mapping Adjustment), and these stations will form the basis for all future scientific and mapping control work.

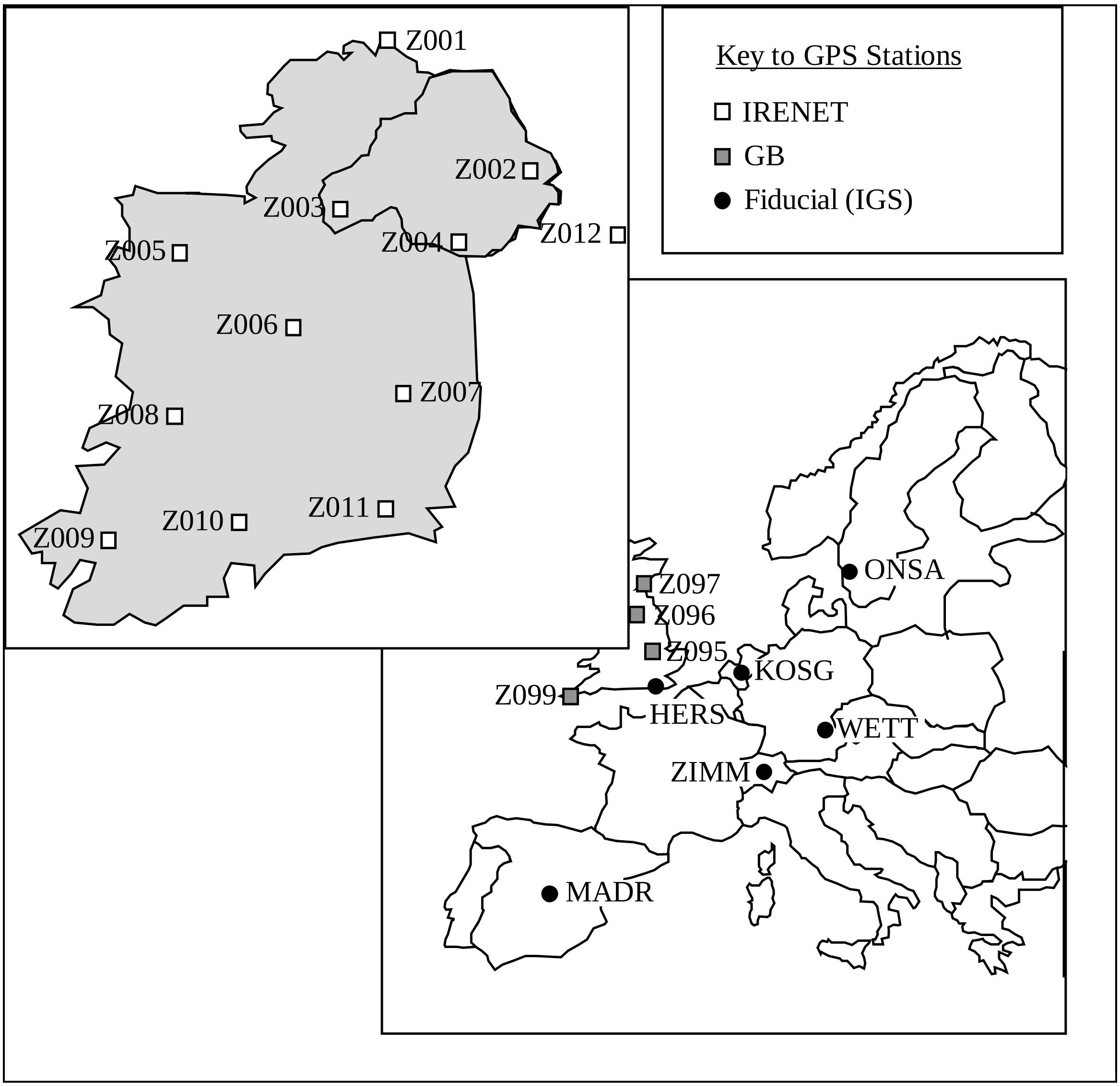


Diagram 11 ***: IRENET Zero Order Stations.***

TECHNICAL DATA

OVERVIEW

National Reference System	Irish Grid
Reference Ellipsoid	Airy Modified
Geodetic Datum	1965 Datum
Vertical Datum	Malin Head
Map Projection	Transverse Mercator
Measurement Unit	International metre

DEFINITIONS AND PARAMETERS

Airy Modified Ellipsoid

All geographical co-ordinates, on which the Irish Grid is based, are expressed in terms of the Airy Modified Ellipsoid, as fixed onto the 1965 Datum. This ellipsoid is based on the Airy Ellipsoid, defined in feet of bar O_1 [3], with a semi-major axis (a) of 20,923,713 feet, and eccentricity squared (e^2) of 0.006 670 540 15. With metrication a conversion factor was agreed between feet of bar O_1 to the International metre of 0.304 800 749 1. The Airy Ellipsoid was reduced by 35 parts per million (ppm) for the Irish reference ellipsoid, resulting in the following, standard parameters:

semi-major axis (a) :	6 377 340.189 m
eccentricity (e^2) :	0.006 670 540 15

The 1965 Geodetic Datum

The Geodetic Datum of the Irish Grid is a derived one based on the positions of ten OSNI primary triangulation stations (1952 adjustment values), and the positions of three OSI primary triangulation stations fixed to their 1965 adjustment values. The 1965 adjustment was a best mean fit to the positions of the Northern Ireland Primary points as adjusted in 1952. The 1952 adjustment was based on the Principal Triangulation positions of three points in Northern Ireland: Knocklayd, Trostan and Divis.

[3] a bar of standard length kept by Ordnance Survey

Malin Head Vertical Datum

This is fixed as Mean Seal Level of the tide gauge at Malin Head, County Donegal. It was adopted as the national datum in 1970 from readings taken between January 1960 and December 1969. All heights on National Grid mapping since then are in International metres above this datum.

Earlier maps (e.g. County Series) used the low water mark of the spring tide on the 8 April 1837 at Poolbeg Lighthouse, Dublin. Initially fixed for County Dublin, it was adopted as the national datum approximately five years later. Heights above this datum were given in (Imperial) feet.

Malin Head datum is approximately 2.7 m above the Poolbeg Lighthouse datum.

THE TRANSVERSE MERCATOR MAP PROJECTION

Ellipsoid	Airy Modified
True Origin	Latitude 53^{o} 30' 00" N
	Longitude 8^{o} 00' 00" W
False Origin	200 kms west of true origin
	250 kms south of true origin
Plane Co-ordinates of True Origin	200 000 E
	250 000 N
Scale Factor on Central Meridian	1.000 035

This projection is used on National Grid Maps at the scales of 1:1 000, 1:2 500, 1:50 000 and 1:250 000.

Other co-ordinate systems and projections used in Ireland are listed in the Appendix A.

NOTATION, SYMBOLS AND STANDARD FORMULAE

All distances are in metres :
Conversion feet to metres: 1 ft = 0.304 800 749 1 m

All angles are in radians

Conversion degrees (decimal) to radians $1^{o} = \frac{\pi}{180}$, or 0.017 453 293 radians

All constants relate to the Irish Grid and Reference Ellipsoid

Standard Notations and Definitions

Notation	**Description, Formulae and Constants**
a	Semi-major axis of ellipsoid. Constant, a = 6 377 340.189 m
b	Semi-minor axis of ellipsoid. Calculated, $b = \sqrt{a^2\left(1-e^2\right)}$ = 6 356 034.447 m
e^2	Eccentricity squared. Constant, e^2 = 0.006 670 540 15 $e^2 = \frac{a^2-b^2}{a^2}$
n	$n = \frac{a-b}{a+b}$
ν	Radius of curvature of the ellipsoid at latitude ϕ perpendicular to a merdian (east - west) $\nu = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$
ρ	Radius of curvature of the ellipsoid at latitude ϕ in the direction of the meridian (north - south) $\rho = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$
η^2	East - west component of the deviation of the vertical, squared $\eta^2 = \frac{\nu}{\rho}-1$
ϕ	Latitude of a point. Can be calculated from E and N of a given point (see page 27).
λ	Longitude of a point (positive (+) east of Greenwich and negative (-) west of Greenwich). Can be calculated from E and N of a given point (see page 27).

H	Height of a point above the ellipsoid. It is necessary to know the geoid - ellipsoid separation at a point, N, so that : $H = (h + N)$ The Airy modified ellipsoid is a good fit to the geoid in Ireland so N can be ignored for most practical purposes, or assumed to be 2.5 m. Use of a geoid model (e.g. OSU91A) is essential for more precise computations.
h	Height of a point above the geoid (MSL). Generally observed by spirit levelling.
ϕ'	Latitude of the foot of the perpendicular drawn from a point on the projection to the central meridian. An iterative process is used to obtain ϕ' as follows: 1. Calculate initial $\phi' = \left(\frac{N - N_0}{aF_0}\right) + \phi_0$ 2. Calculate M (see below). 3. Calculate new value for ϕ' as follows: $\phi'(\text{new}) = \left(\frac{N - N_0 - M}{aF_0}\right) + \phi'_{old}$ 4. Recalculate M using ϕ'(new) in place of ϕ'(initial). 5. If $(N - N_0 - M)$ is close to zero (i.e. < 0.001) then use ϕ'(new). Otherwise recalculate M using ϕ'(new), and repeat steps 3 and 4 above.
ϕ_0	Latitude of the True Origin. Constant, $\phi_0 = 53^{o}$ 30' 00" N
λ_0	Longitude of the True Origin. Constant, $\lambda_0 = 8^{o}$ 00' 00" W
E_0	Grid Easting of the True Origin. Constant, E_0 = 200 000 E
N_0	Grid Northings of the True Origin. Constant, N_0 = 250 000 N
E	Grid Eastings of a point. Can be calculated from ϕ and λ (see page 26).
N	Grid Northings of a point. Can be calculated from ϕ and λ (see page 26).
y	'True' Eastings of a point $y = E - E_0$
x	'True' Northings of a point $x = N - N_0$
F_0	Scale Factor on the Central Meridian. Constant, F_0 = 1.000 035
F	Scale Factor at a point. Can be calculated from E and N or from ϕ and λ (see page 28).
S	True distance between two points on the ellipsoid $S = \frac{s}{F}$

s	Straight line distance between two points on the projection $s = S \times F$
A	True meridional arc $A = b\,[(i)-(ii)+(iii)-(iv)]$ where $(i) = \left\{\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right)(\phi - \phi_0)\right\}$ $(ii) = \left\{\left(3n + 3n^2 + \frac{21}{8}n^3\right)\sin(\phi - \phi_0)\cos(\phi + \phi_0)\right\}$ $(iii) = \left\{\left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right)\sin 2(\phi - \phi_0)\cos 2(\phi + \phi_0)\right\}$ $(iv) = \left\{\frac{35}{24}n^3 \sin 3(\phi - \phi_0)\cos 3(\phi + \phi_0)\right\}$
M	Developed Meridional arc $M = A \times F_0$
C	Convergence - the angle between the grid north and the projected north at a point. Can be calculated from either ϕ and λ or E and N (see page 29).
t	Straight line direction (or chord) between two points on the projection (see page 29).
T	Direction of the line of sight between two points on the ellipsoid as it appears on the projection (direction of the geodesic), (see page 29).
P	Difference in longitude between a point and the longitude of the True Origin (in radians) $P = \lambda - \lambda_0$

Eastings and Northings (E and N) from Latitude and Longitude (ϕ and λ)

$$\mathbf{N} = \boldsymbol{(I) + P^2(II) + P^4(III) + P^6(IV)}$$

where

$$P = (\lambda - \lambda_0)$$

and :

$$(I) = M + N_0$$

$$(II) = \left(\frac{\nu}{2}\sin\phi\cos\phi\right)$$

$$(III) = \left(\frac{\nu}{24}\sin\phi\cos^3\phi\left(5 - \tan^2\phi + 9\eta^2\right)\right)$$

$$(IV) = \left(\frac{\nu}{720}\sin\phi\cos^5\phi\left(61 - 58\tan^2\phi + \tan^4\phi\right)\right)$$

for N_0, F_0, A and other elements - see Standard Formulae above. NB: a and b have been scaled by F_0.

$$\mathbf{E} = \boldsymbol{E_0 + F_0\left[P(V) + P^3(VI) + P^5(VII)\right]}$$

where:

$$P = (\lambda - \lambda_0)$$

and

$$(V) = (\nu\cos\phi)$$

$$(VI) = \left(\frac{\nu}{6}\cos^3\phi\left(\frac{\nu}{\rho} - \tan^2\phi\right)\right)$$

$$(VII) = \left(\frac{\nu}{120}\cos^5\phi\left(5 - 18\tan^2\phi + \tan^4\phi + 14\eta^2 - 58\tan^2\phi\eta^2\right)\right)$$

For E_0,F_0 and other elements- see Standard Formulae above. NB: a and b have been scaled by F_0.

Latitude and Longitude (ϕ and λ) from Eastings and Northings (E and N)

$$\phi = \phi' - y^2(VIII) + y^4(IX) - y^6(X)$$

where

$$(VIII) = \left(\frac{\tan\phi'}{2\rho\nu}\right)$$

$$(IX) = \left(\frac{\tan\phi'}{24\rho\nu^3}\left(5 + 3\tan^2\phi' + \eta^2 - 9\tan^2\phi'\eta^2\right)\right)$$

$$(X) = \left(\frac{\tan\phi'}{720\rho\nu^5}\left(61 + 90\tan^2\phi' + 45\tan^4\phi'\right)\right)$$

For ϕ‘ and other elements see Standard Formulae above. NB: a and b have been scaled by F_0.

$$\lambda = \lambda_0 + y(XI) - y^3(XII) + y^5(XIII) - y^7(XIV)$$

where

$$(XI) = \left(\frac{\sec\phi'}{\nu}\right)$$

$$(XII) = \left(\frac{\sec\phi'}{6\nu^3}\left(\frac{\nu}{\rho} + 2\tan^2\phi'\right)\right)$$

$$(XIII) = \left(\frac{\sec\phi'}{120\nu^5}\left(5 + 28\tan^2\phi' + 24\tan^4\phi'\right)\right)$$

$$(XIV) = \left(\frac{\sec\phi'}{5040\nu^7}\left(61 + 662\tan^2\phi' + 1320\tan^4\phi' + 720\tan^6\phi'\right)\right)$$

For ϕ‘ and other elements see Standard Formulae above. NB: a and b have been scaled by F_0.

True Distance and Grid Distance

The relationship between a distance on a map and the true (natural) distance is expressed as

$$s = S \times F, \quad \text{or} \quad S = \frac{s}{F},$$

where S is the true distance, s is the grid (map) distance and F is the scale factor. Calculating the scale factor of the mid point of a line is sufficiently accurate for most purposes.

For greater accuracy the scale factor at the start point, mid point and end point of a line should be calculated and Simpson's rule applied:

$$\frac{1}{F} = \frac{1}{6}\left(\frac{1}{F_{start}} + \frac{4}{F_m} + \frac{1}{F_{end}}\right)$$

Scale Factor at a Point from ϕ and λ

$$\mathbf{F} \quad = \quad \boldsymbol{F_0\left[1 + P^2(XV) + P^4(XVI)\right]}$$

where:

$$(XV) \quad = \quad \frac{\cos^2\phi}{2}\left(1+\eta^2\right)$$

$$(XVI) \quad = \quad \frac{\cos^4\phi}{24}\left(5 - 4\tan^2\phi + 14\eta^2 - 28\tan^2\phi\eta^2\right)$$

P, η and ν are calculated from the Standard Formulae above. NB: a and b have been scaled by F_0.

Scale Factor at a point from E and N

$$\mathbf{F} \quad = \quad \boldsymbol{F_0\left[1 + y^2(XVII) + y^4(XVIII)\right]}$$

where:

$$(XVII) \quad = \quad \frac{1}{2\rho\nu}$$

$$(XVIII) \quad = \quad \frac{1+4\eta^2}{24\rho^2\nu^2}$$

For ϕ‘ and other elements see Standard Formulae above. NB: a and b have been scaled by F_0.

True Azimuth from Grid Co-ordinates

For a given line A to B :

True Azimuth (A to B) = Grid bearing (A to B) + C - (t-T)

where C is the convergence and (t-T) is the arc to chord correction. The signs (positive or negative) are important, and it is recommended that a diagram be drawn to ensure the corrections are applied in the correct sense (see diagram below and example on page 33).

Convergence C

Convergence C from ϕ and λ*)*

$$\mathbf{C} = \boldsymbol{P(XIX) + P^3\,(XX) + P^5\,(XXI)}$$

where :

$$(XIX) = \sin\phi$$

$$(XX) = \frac{\sin\phi\cos^2\phi}{3}\left(1+3\eta^2+2\eta^4\right)$$

$$(XXI) = \frac{\sin\phi\cos^4\phi}{15}\left(2-\tan^2\phi\right)$$

P and η are calculated from the Standard Formulae above. NB: a and b have been scaled by F_0 .

Convergence C from E and N

$$\mathbf{C} = \boldsymbol{y(XXII) - y^3(XXIII) + y^5(XIV)}$$

where :

$$(XXII) = \frac{\tan\phi'}{\nu}$$

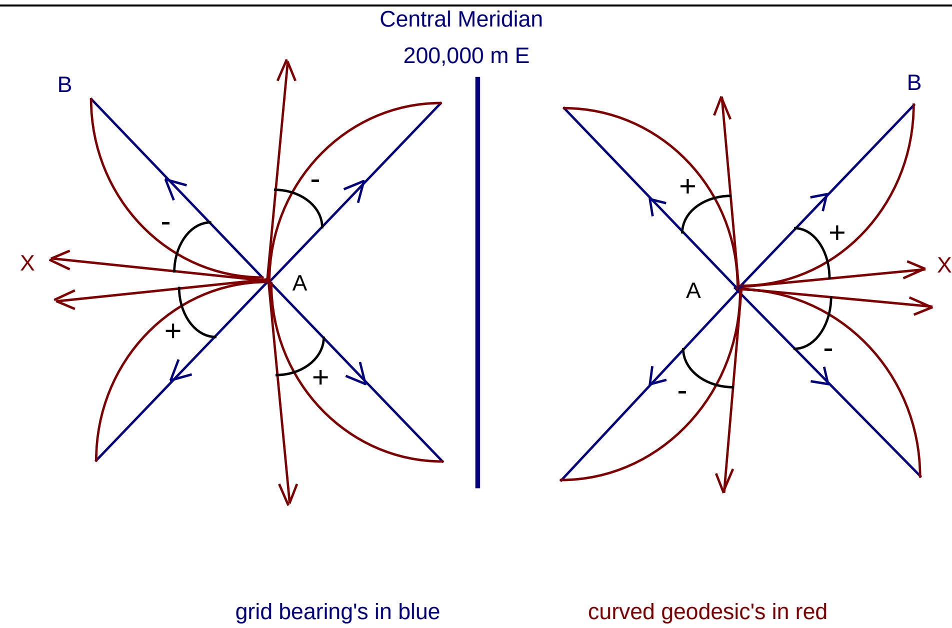
$$(XXIII) = \frac{\tan\phi'}{3\nu^3}\left(1+\tan^2\phi'-\eta^2-2\eta^4\right)$$

$$(XXIV) = \frac{\tan\phi'}{15\nu^5}\left(2+5\tan^2\phi'+3\tan^4\phi'\right)$$

For ϕ‘ and other elements see Standard Formulae above. NB: a and b have been scaled by F_0 .

(t-T) Correction

As a guide, the following diagram shows geodesic's in each quadrant of the compass, either side of the central meridian, and the signs of the (t-T) correction. The example computation on page 33 shows a specific case.



The sign of (t-T) correction's

(t -T) from E and N

The following formulae are for correcting real world directions to the plane grid. If 1 and 2 are the terminals of the line, then :

$$(t_1 - T_1) = (2y_1 + y_2)(N_1 - N_2)(XXV)$$

$$(t_2 - T_2) = (2y_2 + y_1)(N_2 - N_1)(XXV)$$

where :

$$(XXV) = \frac{1}{6\rho\nu}$$

For ϕ' use the iterative process described in the Standard Formulae section above, but use the mean value of the Northings at points 1 and 2. Other elements may then be computed as per the Standard Formulae above. NB: a and b have been scaled by F_0 .

Note: The answer is in radians. Convert to seconds multiplying by $\frac{1}{\sin 1''}$

EXAMPLE COMPUTATIONS

(NB in all the following calculations a and b have been scaled by F_0)

Example 1 :
Eastings and Northings (E and N) from Latitude and Longitude (ϕ and λ)

a) OSO

Latitude, ϕ = 53° 21' 50".5441 N
Longitude, λ = 06° 20' 52."9181 W

(i) = -0.0023769279512547
(ii) = 3.46130364893521E-06
(iii) = 2.07542844095922E-08
(iv) = -3.7575342614459E-11
A = -15130.2334883926

n = 0.00167322034796603
ν = 6391304.32563859
ρ = 6376057.7025783
η^2 = 0.00239123040779954

P = 0.0288322666779889

I = 234869.766511607
II = 1530208.55423369
III = 145902.021809838
IV = -21888.4241245653
V = 3813874.22135243
VI = -182416.601340103
VII = -98720.1961981523

Eastings, E = 309 958.2645
Northings, N = 236 141.9291

b) Howth

Latitude, ϕ = 53° 22' 23".1566 N
Longitude, λ = 06° 04' 06".0065 W

(i) = -0.0022185529826205
(ii) = 3.23236200794504E-06
(iii) = 1.93673429493052E-08
(iv) = -3.508539608726E-11
A = -14122.1151195475

n = 0.00167322034796603

ν = 6391307.56716596
ρ = 6376067.40396709
η^2 = 0.00239021362750735

P = 0.0337139118714679

I = 235877.884880453
II = 1530063.82361988
III = 145771.556495997
IV = -21902.7266261809
V = 3813065.2161548
VI = -182570.814483949
VII = -98684.7977252937

Eastings, E = 328 546.3442
Northings, N = 237 617.1863

Example 2:
Latitude and Longitude (ϕ and λ) from Eastings and Northings (E and N)

a) OSO

Eastings, E = 309 958.26
Northings, N = 236 141.93

(i) = -0.00217658458592961
(ii) = 3.17165352619869E-06
(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53° 22' 31".6984

n = 0.00167322034796603
ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

VIII = 1.6506139597244E-14
IX = 3.49963747125649E-28
X = 1.02020139910188E-41

XI = 2.62270777207874E-07
XII = 4.94598619656873E-21
XIII = 1.75890124784217E-34
XIV = 7.52067321539881E-48

Latitude, ϕ = 53° 21' 50".5442 N
Longitude, λ = 06° 20' 52".9183 W

b) Howth

Eastings, E = 328 546.34
Northings, N = 237 617.19

(i) = -0.00194487640605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = 53^o 23'19".4228

n = 0.00167322034796603
ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960846801

VIII = 1.65140697291118E-14
IX = 3.50307748554147E-28
X = 1.02190562389051E-41

XI = 2.6235225124939E-07
XII = 4.95126057378679E-21
XIII = 1.76207752818572E-34
XIV = 7.53971553553849E-48

Latitude, ϕ = 53^o 22' 23".1567 N
Longitude, λ = 06^o 04' 06".0067 W

Example 3 :
Scale Factor from Latitude and Longitude

a) OSO

Latitude, ϕ = 53^o 21' 50".5441 N
Longitude, λ = 06^o 20' 52."9181 W

ν = 6391304.32563859
ρ = 6376057.7025783
η^2 = 0.00239123040779954

(XV) = 0.178468264698499
(XVI) = -0.012261592793434

Scale Factor = 1.000 183 36

b) Howth

Latitude, ϕ = 53^o 22' 23".1566 N
Longitude, λ = 06^o 04' 06".0065 W

ν = 6391307.56716596
ρ = 6376067.40396709
η^2 = 0.00239021362750735

(XV) = 0.178392196865632
(XVI) = -0.0122766174316604

Scale Factor = 1.000 237 76

Example 4:
Scale Factor from Eastings and Northings

a) OSO
Eastings, E = 309 958.26
Northings, N = 236 141.93

(i) = -0.00217658458592961
(ii) = 3.17165352619869E-06
(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53^o 22’ 31”.6984

ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

(XVII) = 1.22695082389524E-14
(XVIII)= 2.53299951778783E-29

Scale Factor = 1.000 183 36

b) Howth
Eastings, E = 328 546.34
Northings, N = 237 617.19

(i) = -0.00194487640605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = 53^o 23’19”.4228

ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960847

(XVII) = 1.22694718188187E-14

(XVIII)= 2.53296954941824E-29

Scale Factor = 1.000 237 76

Example 5 :
True Distance from Grid Distance

a) Line OSO to Howth

Grid Distance OSO to Howth = 18 646.53 m

1st point, OSO	Eastings	= 309 958.26 E
	Northings	= 236 141.93 N
2nd point, Howth	Eastings	= 328 546.34 E
	Northings	= 237 617.19 N
mid point	Eastings	= 319 252.30 E
	Northings	= 236 879.56 N

(i) = -0.00206073049599361
(ii) = 3.00397954545871E-06
(iii) = 1.79858170097256E-08
(iv) = -3.2602179751047E-11
A = -13117.5119880953
ϕ' = 53^{o} 22’ 55”.5606

ν = 6391310.78767229
ρ = 6376077.04245292
η^2 = 0.00238920344248461

(XVII) = 1.22694900282341E-14
(XVIII)= 2.53298453304199E-29

Scale Factor at mid point= 1.00 02 09 50
True distance (i) (using SF at mid point) = 182 642.625 m

Applying Simpson’s rule

Scale Factor from first point (see example 4b)	= 1.000 183 36
Scale Factor from second point (see example 4a)	= 1.000 237 76
Scale Factor from Simpson’s Rule	= 1.000 209 85

True Distance (ii) from Simpson’s Rule SF = 18 642.619 m

Example 6:
True Azimuth from Grid Coordinates
a) Line OSO to Howth

2nd Point	**Howth**	E_2	328 546.34	N_2	237 617.19

1st Point	**OSO**	E_1	309 958.26	N_1	236 141.93
		E_2-E_1	18 588.08	N_2-N_1	1 475.26

Grid Bearing 1 - 2 (a1) = $\arctan\left[\dfrac{(E_2 - E_1)}{(N_2 - N_1)}\right]$ = 85° 27' 43".8474

Grid Distance 1 to 2 = $\left[\dfrac{(E_2 - E_1)}{\sin\alpha_1}\right]$ = 18 646.531m

= $\left[\dfrac{(N_2 - N_1)}{\cos\alpha_1}\right]$ = 18 646.531m (check)

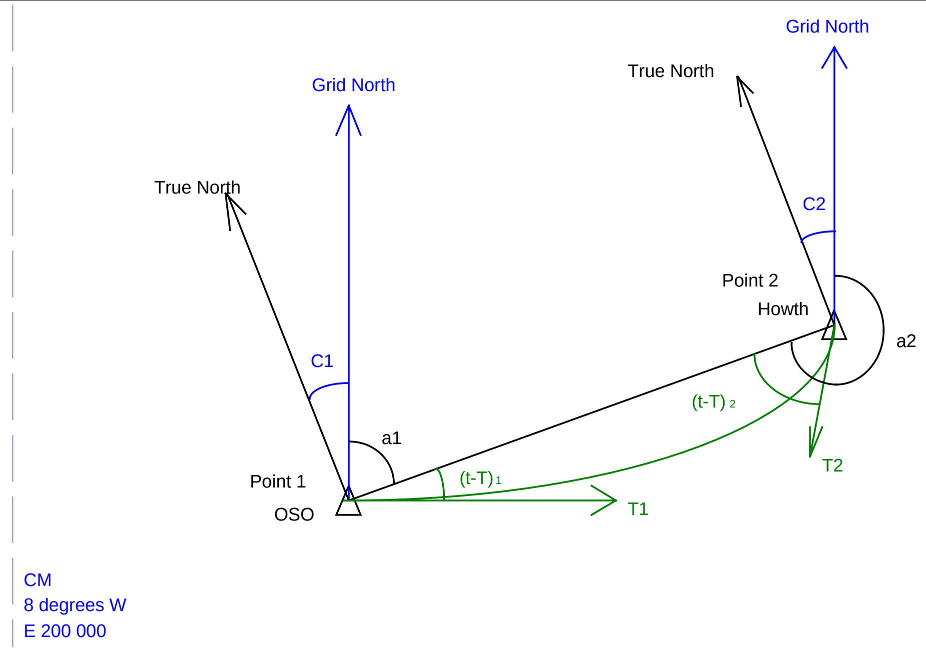


***Diagram 6 :** The relationship between True Azimuth, Grid Azimuth, Convergence and (t-T) correction for Example 6.*

True Azimuth$_1$ = $a1 + C1 - (t - T)"_1$
True Azimuth$_2$ = $(a1 + 180) + C2 - (t - T)"_2$

Convergence C at a point
Point 1 OSO
Easting, E = 309 958.26
Northing, N = 236 141.93

(I) = -0.00217658458592961
(ii) = 3.17165352619869E-06

(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53^{o} 22' 31".6984

ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

XXII = 2.10488601181288E-07
XXIII = 4.82209555816333E-21
XXIV = 1.75556049978879E-34

Convergence, C1 at OSO = 01^{o} 19' 32".6690

Point 2 Howth
Eastings, E = 328 546.34
Northings, N = 237 617.19

(I) = -0.00194487610605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = 53^{o} 23' 19".4228

ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960846801

XXII = 2.10590196160653E-07
XXIII = 4.82742602059825E-21
XXIV = 1.75873870923192E-34

Convergence, C2 at Howth = 1^{o} 33' 01".5981
(t-T)
Eastings and Northings of points 1 and 2 as above.
ν = 6391310.78767229
ρ = 6376077.04245292

XXV = 4.08983000941E-15
y_1 = 109958.26
y_2 = 128546.34
N1 and N2 as above

$(t-T)_1$ = - 0".4337
$(t-T)_2$ = + 0".4568

Results:

True Azimuth$_1$		**True Azimuth$_2$**	
a1	85° 27’ 43”.8474	a2	85° 27’ 43”.8474
+C1	1° 19’ 32”.6690	+180°	180° 00’ 00”.0000
-(t-T)$_1$	-0”.4337	+C2	1° 33’ 01”.5981
		-(t-T)$_2$	+ 0”.4568
True Azimuth$_1$	86° 47’ 16”.9501	True Azimuth$_2$	267° 00’ 44”.9887

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[5] ORDNANCE SURVEY, 1967. *The History of the Retriangulation of Great Britain 1935-1962.* H.M.S.O., London.

[6] ANDREWS, J.H., 1975. *A Paper Landscape, The Ordnance Survey in Nineteenth-Century Ireland.* Clarendon Press, Oxford.

[7] TAYLOR, W.R. (Lt. Col.), 1967. *An outline of the re-triangulation of Northern Ireland.* H.M.S.O., Belfast.

[8] ASHKENAZI, V., CRANE, A.S., PREISS, W.J., WILLIAMS, J.W., 1980. *The 1980 Readjustment of the Triangulation of the United Kingdom and the Republic of Ireland OS(SN)80*. Ordnance Survey (GB) Professional Paper New Series No 31, Southampton.

APPENDIX A

OTHER SYSTEMS USED IN IRELAND

NAME	ELLIPSOID	DATUM NAME	DESCRIPTION
Ireland 1965	Airy Modified	Ireland 1965	See Main Report
OSGB70(SN)	Airy	Herstmonceux	The Irish network (1965) was adjusted to OSGB70(SN)[4] junction stations, incorporating additional cross-channel and astronomic observations.
ED50	International Hayford	Helmert Tower, Potsdam	The Irish network (1965) was adjusted to OSGB70(SN) junction stations. ED50 values of Irish triangulation stations obtained by 3D cartesian transformation, derived at Herstmonceux. Unknown stations may be obtained by deriving 3D transformations at nearest three or four surrounding triangulation stations, and applying these to the unknown point.
OS(SN)80	Airy	Herstmonceux	Combined adjustment of Irish and GB terrestrial observations with positions of some stations determined by weighted TRANSIT doppler observations. [5]
ED87	International Hayford	As per ED50	A full readjustment of European terrestrial networks scaled and controlled by space techniques. Observations from Ireland are based on those used in the OS(SN)80 adjustment.
WGS84	WGS84	Earth-centred co-ordinate system	WGS84 is defined by the positions of some 1591 stations observed using TRANSIT doppler observations. Because of this WGS84 is accurate to about 1-2 metres, globally. WGS84 co-ordinates of Irish primary triangulation stations were derived by transformation resulting from TRANSIT doppler observations in the 1980's.

[4] Ashkenazi V., Cross P., Davies M.J.K., Proctor D.W. *The Readjustment of the Retriangulation of Great Britain, and its Relationship to the European Terrestrial and Satellite Networks.* Ordnance Survey Southampton, 1972.
[5] Ashkenazi V., Crane A., Preiss W., Williams J. *The 1980 Readjustment of the Triangulation of the United Kingdom and the Republic of Ireland OS (SN) 80*. Ordnance Survey (Southampton) Professional Paper, New Series No 31.

ETRF / ITRF	GRS80	Earth-centred co-ordinate reference frame.	The European Terrestrial Reference Frame (ETRF) is a regional subset of the International Terrestrial Reference Frame (ITRF), which is derived by high precision satellite and space observations at many global geodetic observation facilities. It is thought to be accurate to a few centimetres, which is enough to detect physical movement of stations. Thus annual realisations of positions are possible in this framework, resulting in the necessity of time stamping.
ETRS89	GRS80	Earth-centred co-ordinate system.	Positions of points are referred to as being in the ETRF (or ITRF), but are expressed as part of a co-ordnate system which is time stamped at particular epochs. Thus ETRS89 is fixed to the 1989 epoch, and is more formally known as 'ETRS89 (epoch 1989.00)'.

REFERENCE ELLIPSOIDS

REFERENCE ELLIPSOID	**DEFINING PARAMETERS**	**COMMENTS**
International (Hayford)	a = 6 378 388.0m e^2 = 0.006 722 670 022 33	
Airy	a = 6 377 563.3964 b = 6 356 256.9096 e^2 = 0.006 670 540 000 12	
WGS84	a = 6 378 137.000 e^2 = 0.006 694 379 9	There are other defining parameters.[6]
GRS80	a = 6 378 137.000 b = 6 356 752.314 1 e^2 = 0.006 694 380 022 90	Strictly speaking GRS80 is defined by many more constants, however these are not of direct interest in this instance.[7]

[6] Defence Mapping Agency, 1987. *Department of Defense World Geodetic System 1984. Technical Report (and supplements).* DMA TR-8350.2, USA.
[7] Moritz, H., 1988. 'G*eodetic Reference System 1980'*. Bulletin Geodesique, 1988 Volume 62 No 3, Paris.

Appendix A (contd)

PROJECTIONS USED IN IRELAND

Bonne Projection

Projection :	Simple conic projection with one standard parallel
Co-ordinates :	Rectangular plane co-ordinates
Units :	Feet of bar O_1
Central Meridian :	08° 00’ 00” W
Standard Parallel :	53° 30’ 00” N
Origin :	Intersection of CM and SP
Radius of mean parallel :	15 516 209.8 feet of Bar O_1
Ellipsoid :	Airy
Limits of System :	Ireland
Properties :	Not conformal
Map scales :	1 inch, 1/2 inch and 9 mile to one inch

Lambert Conformal Conic

Projection :	Conformal conic with two standard parallels
Central meridian :	08° 00’ 00” W
Standard Parallels :	52° 40’ 00” N and 55° 20’ 00” N
Ellipsoid :	International Hayford
Properties :	Conformal
Map scales :	Aeronautical Charts, 1:1 000 000 and 1:500 000

Cassini Projection

Projection :	Cylindrical projection with line of contact along chosen central meridian
Co-ordinates :	Rectangular spherical co-ordinates
Units :	feet of Bar O1
Central Meridian :	Longitude of Initial points (see Appendix B)
Origin :	Different origin for each county (see Appendix B)
Ellipsoid :	Sphere used
Limits of system :	Separate county projections to limit scale errors
Properties :	Not conformal
Map scales :	County Series 6 inch and 25 inch to one mile

APPENDIX B

ORIGINS OF COUNTY SERIES CASSINI PROJECTIONS.

County	Initial Point	Latitude N	Longitude W	Meridional Distance (m)
Donegal	Letterkenny Church Spire	54° 57’ 02”.84	7° 44’ 18”.53	
Monaghan	Monaghan Church Tower	54° 14’ 53”.06	6° 58’ 07”.38	
Sligo	Cooper’s Observatoryy	54° 10’ 30”.17	8° 27’ 24”.93	19,702,286.200
Mayo	Castlebar Church Tower	53° 51’ 15”.07	9° 17’ 58”.72	
Leitrim	Ck-on- Shannon Church Spire	53° 56’ 48”.43	8° 05’ 37”.74	
Cavan	Cavan Church Spire	53° 59’ 37”.21	7° 21’ 35”.31	
Louth	Dundalk Church Spire	54° 00’ 29”.73	6° 24’ 03”.43	
Meath	Wellington Testimonial	53° 33’ 06”.13	6° 47’ 35”.17	
Westmeath	Mullingar Church Spire	53° 31’ 27”.91	7° 20’ 20”.88	19,464,692.554
Longford	Longford Church Spire	53° 43’ 51”.35	7° 47’ 57”.85	
Roscommon	Roscommon Church Spire	53° 37’ 43”.73	8° 11’ 24”.87	
Dublin	Dublin Observatoryy	53° 23’ 13”.00	6° 20’ 17”.51	19,414,493.354
Offaly	Tullamore Church	53° 16’ 17”.64	7° 28’ 50”.91	
Galway	Galway Church Spire	53° 16’ 20”.49	9° 03’ 11”.60	
Clare	Ennis Church Tower	52° 50’ 44”.07	8° 58’ 51”.44	19,216,822.850
Laois	Port Laoise New Church Spire	53° 02’ 01”.31	7° 18’ 10”.24	
Wicklow	Lugnaquilla	52° 57’ 59”.94	6° 27’ 50”.23	19,261,029.515
Carlow	Mount Leinster	52° 37’ 03”.47	6° 46’ 46”.96	
Kilkenny	Kilkenny Church Spire (St. Mary’s)	52° 39’ 04”.41	7° 15’ 07”.00	
Tipperary	Cashel Church Spire	52° 30’ 53”.48	7° 53’ 06”.71	
Limerick	Rice’s Monument	52° 39’ 27”.61	8° 37’ 40”.72	
Kerry	Tralee Church Spire	52° 16’ 13”.61	9° 42’ 11”.99	19,006,846.490
Cork	Mount Hillary	52° 06’ 34”.72	8° 50’ 20”.50	18,948,141.837
Wexford	Forth Mountain	52° 18’ 56”.00	6° 33’ 42”.37	19,023,314.460
Waterford	Knockanaffrin	52° 17’ 18”.46	7° 34’ 53”.71	19,013,422.930
Kildare	Kildare Round Tower	53° 09’ 27”.68	6° 54’ 41”.86	